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# DO SATURATED ASTROPHYSICAL MASERS EXHIBIT COHERENCE?

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## ABSTRACT

Goldreich and Kwan's (1974) recent evaluation of the inadequacy of the traditional explanation for the relationship between saturated astrophysical maser line intensity and linewidth is supported. However, their proposal depends critically on the stability of their solutions having Gaussian statistics, and on the total size and temperature of the masing regions. They required that the maximum stimulated emission rate (W) for the masing levels is much less than the infrared spontaneous emission rate (A) from higher levels into these levels, in order that trapped infrared radiation can explain the narrowest linewidths. However, their solutions are not stable in saturated masers. An earlier proposal for stable coherent (pulsed) radiative outputs from saturated astrophysical masers, most likely class I OH masers and perhaps $H_2O$ masers is then expanded, which seems to be in qualitative agreement with current observations. It predicts linewidths much narrower than those produced by Doppler broadening. The model may also explain some of the differences observed between main line OH sources and $H_2O$ masers. Furthermore, it predicts total OH and $H_2O$ maser sizes roughly those obtained by the VLBI measurements of these masers, and thus much smaller than those previously derived by other authors for the actual rather than the apparent maser size.

## I. Introduction

Recently, Goldreich and Kwan (1974) have given some arguments which this author also believes to be correct as to why the traditional description of astrophysical maser line formation is inadequate to explain the linewidths observed for OH, and possibly $H_2O$ masers. These arguments can be summarized as follows, and we will concentrate on how the points apply to OH observations:

1). The traditional theory allows linewidths to narrow from their Doppler widths inversely proportional to the square root of the logarithm of the gain during unsaturated or exponential growth of the line intensity.

2). OH masers are believed to be highly saturated given the extremely high brightness temperatures observed, even if the apparent sizes of the masing clouds are as much as 100 times smaller than their actual physical size, and given their high degree of polarization.

3). Once saturation sets in the traditional theory (Litvak 1973a) demands that the lines should rather quickly rebroaden to approach their Doppler widths ($\Gamma_D$), or $\Gamma_D/\sqrt{2}$ or so for spherical geometry.

4). According to the best data on OH masers by Ronnang (1972) (Goldreich and Kwan cite the less accurate older data by Barrett and Rodgers (1966)) the linewidths vary in terms of equivalent Doppler temperatures from 7° K to 130° K for the lines that seem well individuated by his computer program, with a substantial proportion falling in the range under 40° K.

5). Since we do not believe that gas temperatures in these relatively dense clouds undergoing masing are this low or even only a factor of 2 or 3 greater for the narrowest lines, we need an alternate theory of the relationship between line intensity after saturation, and linewidth.

The purpose of this paper is to greatly elaborate on the proposal for such an alternate theory outlined by the author earlier (1974a). First, however, we must comment on the requirements and assumptions of the counter proposal put forward by Goldreich and Kwan. Until recently, this author believed that while the evidence favored his theory for saturated astrophysical masers, the counter-proposal of Goldreich and Kwan was possibly correct if it could be shown that coherence was impossible to establish in astrophysical masers. However, a relatively simple argument proposed by Isaksson (1973) demonstrates that the Gaussian statistics of the radiation, assumed by Goldreich and Kwan, is destroyed with further amplication after saturation in a maser. This is similar to the well-known change in statistics of a single mode laser operating above threshold, where the amplitude fluctuations are stabilized, as the amplitude continues to grow. (This argument showing the evolution of the probability density was presented by Isaksson for homogeneously broadened lines, but holds separately for each Fourier component of a inhomogeneously broadened line with stationary statistics.)

Equally important for altering the nature of the radiation, are the correlations which develop between the field and matter variables as Appendix B of Goldreich, et. al. (1973a) begins to discuss for saturated masers. However, they did not seem to realize the likelihood that these correlations represented the first stages of the evolution of the field into a coherent wave. Thus a field with stationary statistics seems unstable in a saturated maser just as is a field with Gaussian statistics. By contrast, the solutions utilized in the author's earlier model are stable (see Barone 1972). These two theoretical conjectures have been completely borne out in the one-dimensional case by numerical maser calculations recently completed by Friedberg and Rosen (1976). Thus it becomes unlikely that Goldreich and Kwan's proposal could explain line shapes in saturated masers, and most other theories for this regime are thrown into doubt. The details of this argument will be presented in Section II, along with a discussion of the limitations imposed upon Goldreich and Kwan's theory even if their solution was stable.

Section III will provide the theoretical background for our alternate model for astrophysical masers involving coherent Self-Induced Transparency wave trains, the stages of which will be developed in Section IV. Finally, Section V will discuss in detail how the existing evidence bears on this model versus other previous models.

## II. Critique of the Goldreich and Kwan Proposal

As Goldreich and Kwan provide a nice review of the traditional approach to maser models using a rate equations for the level populations and an equation of radiative transfer for the intensity instead of Maxwell's equation, there is no need to repeat that here. It is important, though, to note as they do, that the homogeneous broadening ($\Gamma$) of masing levels due to collisions is much less than the Doppler width ($\Gamma_D$). For example, in the case of OH, typical parameters are: $N_{H_2} = 10^8$ cm$^{-3}$, $N_{OH} = 10^2$ cm$^{-3}$, and $T = 10^{2°}$ K yield $\Gamma = T_2^{-1} \approx 10^{-2}$ s$^{-1}$ and $\Gamma_D = T_2^{*-1} \approx 10^3$ (Goldreich and Keeley 1972). Even for class 2b infrared pumped OH sources, an estimate of the pump rate $\lambda$ is that $\lambda \approx 10^{-1}$ s$^{-1}$, i.e. $T_2 = 10^1$ s in this case (Litvak 1969). They proceed to claim that while caution is required when using the rate equation approach, this approach is justified in this case for two reasons. The first is that the radiation field in the sources has Gaussian statistics, which, of course, they can only infer indirectly from measurements made on the earth. The second is that its spectral line width is much greater than the maximum stimulated emission rate (W) calculated assuming a certain real size for the masing clouds which may be much too large, as we will show in Section IV. This would cause Goldreich and Kwan to greatly underestimate W. Regardless of this possibility, however, this second point cannot be used to justify the assumption that Gaussian statistics will persist. As we shall explain shortly, the relevant comparison is not between $\Gamma_D$ and W, but between W and $\Gamma$ in order to determine whether the statistics of the radiation will remain Gaussian.

Aside from this, however, Goldreich and Kwan's approach to explaining the very narrow linewidths among the strongest Class I OH sources is critically dependent on the estimates for $W_{max}$, maser geometry, and kinetic temperature of the gas. Even if a mechanism such as that proposed by Goldreich and Kwan would allow the linewidth to continue to narrow indefinitely after saturation proportional to $(\ell n\, G)^{-\frac{1}{2}}$, where G is the gain, the existing data would still be difficult to explain if they have underestimated W. That is because although they specify more than one criteria to make their mechanism operative, one is that each infrared transition from a masing level satisfied $A/[h\nu/KT - 1] > W$, where A and $\nu$ are respectively the spontaneous emission rate and transition frequency of the trapped I R line. Given their estimate of A for main line OH transitions, this criteria would require $W << .8\ sec^{-1}$ for $T = 600°K$, even less at lower temperatures, and that the development of coherence postulated in this paper does not occur, for this would greatly increase the maximum W. If $W_{max} \sim .8\ sec^{-1}$ implying no highly coherent lines, then Goldreich and Kwan's mechanism may be adequate to explain the observed data if kinetic temperatures for the masing gases are $\leq 200°K$, since the maximum gain is about $e^{25}$ and the narrowest linewidths are about 8°K. But as we have said, the relevance of their mechanism becomes highly dependent on their assumption that Gaussian and stationary statistics are maintained for the strongest lines.

The alternative, then, to both Goldreich and Kwan's and Litvak's approach to maser models in terms of explaining the relationship between linewidth and intensity, is not to make their assumption of Gaussian statistics for the radiation at all intensities, especially when the masers become saturated. This assumption is not even justified in Litvak's early paper (1970b) where he derives the radiative transfer equation from the density matrix equations for a gas of two level molecules in conjunction with Maxwell's equation. Yet it is critical when Litvak factors products of the electric field (E), the dipole moment (P), and the population inversion ($\Delta$) without considering the possibility of correlations between them. Goldreich, Keeley, and Kwan (1973a) correctly demonstrate that assuming stationary statistics (a less specific requirement than Gaussian statistics) requires $\Delta$ to be a constant independent of time in an unsaturated maser.

As their Appendix B shows, however, this is true only to zeroth order in a perturbation series of powers of $W/\Gamma$. Their use of second order perturbation theory shows that at saturation the last term in the expression (B10) for the autocorrelation function of $\Delta$ that represents the change in the autocorrelation time from infinity to $T_2$ becomes as important as the constant part of $\Delta$.

What Goldreich, et al. did not seem to realize is that this term also represents the onset of correlations between the matter variable $\Delta$ and field variable E. In terms of Fourier components the term of interest in equation (B10) is related to the correlation $\langle E(w)E^*(w + \nu)\, \Delta(\nu)\rangle$, which is non-zero even in first order perturbation theory, as is shown in Appendix A of this paper. This is non-zero for $\nu \neq 0$ because it represents a correlation between $\Delta$, and E at two different frequencies, and their expression (B10) seems to be in error for not containing f at two different frequencies. (Again, see Appendix A.) This and similar correlations indicate coherence developing, since they occur on a much longer time scale $T_2$ than the noisy field fluctuations (time scale $T_2^*$) in an inhomogeneously broadened maser. Of course, an exact comparison of the coherent versus incoherent parts of $\Delta$ at saturation cannot be derived using perturbation theory, since the perturbation series diverges at saturation ($W = \Gamma$). Proceeding beyond a second order calculation in $W/\Gamma$ indicates as Goldreich, et al. guessed that the autocorrelation time for $\Delta$ becomes $\sim W^{-1}$ after saturation.

While much additional work has yet to be done on the problem of a three-dimensional saturated maser, in one-dimension and probably in three-dimensions it is clear that once correlations like those between $\Delta$ and E become important, then the products of $\Delta$, E, and P that appear in Litvak's (1970b) density matrix equations (7) cannot be factored into only paired correlations at all. In general only Gaussian variables can be factored pairwise. Certainly no one has justified this factorization thus far. Clearly coherent effects that interconnect different frequencies of the electric field through the molecular variables must be taken into account, and the saturated maser problem cannot be solved assuming all frequencies act independently of each other the way Goldreich, et al., and Litvak have done. The development of correlations at saturation, as has been verified numerically in one-dimension by Friedberg and Rosen (1976), throws all of the results obtained for saturated three-dimension masers by Goldreich, et al., and Litvak in doubt.* Furthermore, these coherent effects make likely this author's earlier proposal that the solution for the radiation field in a saturated maser of any dimensionality is closely approximated by the only stable analytical solution known to exist for the coupled Maxwell-Block equations, namely the coherent phenomenon known as self-induced transparency (SIT). More precisely, the numerical calculations seem to yield somewhat randomly fluctuating SIT pulses.

This alternative becomes plausible from an examination of the literature of the last seven years on the distortionless propagation of coherent light through a two-level medium (SIT), discovered by McCall and Hahn (1967). For a single pulse, SIT has been clearly demonstrated experimentally and theoretically when the pulse width $\tau = \left(\frac{\mu E_{max}}{2\hbar}\right)^{-1}$ is much less than $T_2 = \Gamma^{-1}$, the time characteristic of homogeneous broadening. $E_{max}$ is the maximum of the electric field envelope. The remarkable fact about SIT is that it occurs independently of the relationship between $\tau$ and $T_2^*$ (which characterizes Doppler broadening).

* This is because the non-linear effects described above should couple electric field components Fourier analyzed into different three-dimensional wave vectors $\vec{k}$, just as different frequency components become coupled for one-dimensional propagation.

Even allowing for drastically different apparent and real masing cloud sizes, electric fields of as high as at least $10^{-7}$ esu, or more likely $10^{-6}$ esu are believed to exist on the outer perimeters of the OH clouds (Townes 1971). Thus $\tau^{-1}$ would be at least $10^2\ s^{-1}$, when $E_{max} = 10^{-7}$ esu, and thus here $\tau << T_2$ by several orders of magnitude. It is possible, then, given the parameters often assumed for the strongest OH astrophysical masers, that some form of a SIT wave would be the equilibrium state near the cloud perimeter. If the radiation travelling through the cloud has a sufficient interaction distance with the OH medium after saturation occurs (i.e. when $W \geq \Gamma$), which is required to achieve this equilibrium state, then the radiation should be emitted as at least apartially coherent wave, the form of which will be described more fully in the next section. This wave would not have stationary statistics, as the Fourier components of the electric field at different frequencies would be correlated. Furthermore, the stimulated rate of emission (W) which it imposes on the molecules would roughly equal its line width. From these properties of SIT wave trains we can see how crucial the assumption of Gaussian statistics is to much of the reasoning about maser models that has occurred in the recent literature. We will examine this influence more extensively later.

## III. Solution of the Saturated Maser Equations

### a) The Pumped Coherent Solution

Once we realize that the assumption of Gaussian statistics for very strong astrophysical masers can no longer be made we are forced to solve the density matrix equations that Litvak (1970b) began with in their full non-linear complexity simultaneously with Maxwell's equation. The density matrix equations for a two level system differ from the rate equations in that there is a third equation for the time dependence of the off-diagonal matrix element $\rho_{12}$ not included in the rate equations, which represents the polarization of the molecules induced by the electric field. This field must in turn be self-consistent with the incident field plus the additional field radiated by the average macroscopic dipole moment of the medium. This self-consistency is insured by Maxwell's equation. The two diagonal density matrix elements are the two level population densities. The density matrix equations in the interaction representation used by Litvak were similar to the following:

$$\dot{\rho}_{11} = i^{-1} \frac{\mu}{\hbar} E (\rho_{21} - \rho_{12}) - \Gamma' \rho_{11} + R_1 \quad , \tag{1}$$

$$\dot{\rho}_{22} = - i^{-1} \frac{\mu}{\hbar} E (\rho_{21} - \rho_{12}) - \Gamma' \rho_{22} + R_2 \quad , \tag{2}$$

and

$$\dot{\rho}_{12} = i^{-1} \frac{\mu E}{\hbar} (\rho_{22} - \rho_{11}) - \Gamma \rho_{12} \quad , \tag{3}$$

where $\mu$ is the magnitude of the dipole matrix element connecting the two states, E is the total electric field, and $R_i$ are the two pump rates into the two levels. These equations take account of the short term fluctuation and oscillations that would occur at about the rate of oscillation of the electric field, that the rate equations omit.

The notation used in the field of Self-Induced Transparency usually follows that traditional from discussion of the Bloch equations, to which the density matrix equations written above can be reduced when the $R_i$'s are omitted. They are in Crisp's (1969) notation:

$$\dot{X} = -\delta y - x/T_2 \quad , \tag{4}$$

$$\dot{Y} = \delta x + \frac{\mu E}{\hbar} Z - y/T_2 \quad , \tag{5}$$

and

$$\dot{Z} = -\frac{\mu}{\hbar} Ey - (Z-Z_o)/T_1 \quad . \tag{6}$$

$T_1$ and $T_2$ are two phenomenological decay constants ($T_2$ we already have discussed) which represent the effects of collisions and other processes in causing the population inversion and the polarization, respectively, to relax toward their equilibrium values. $Z = N_2 - N_1$, X and Y are the components of the polarization that are respectively in phase and out of phase with the electric field, and $\delta = \omega_o - \omega$ , the amount the frequency of the field is off-resonance. The pumping can later be added on again, but we will see that it is not very important how this is done. We note that $T_1$ will tend to be even larger than $T_2$, since many collisions will not induce a transition between levels, whereas all collisions will dephase the microscopic dipole moments to some extent. Since we will be interested in solving the Bloch equations with pumping, we will not have to worry about the population inversion relaxing to $Z_o$, and thus we ignore $T_1$ (let $T_1 = \infty$) and focus our attention on the shorter of the two decay constants $T_2$, since this will tend to cause the greater broadening of the spectral lines anyway. By applying the Bloch equations for a two level system to the OH or $H_2O$ maser problem, we are ignoring the effects of level degeneracy. It has been shown, however, that at least for transitions involving changes in integer angular momentum of 0 or 1, level degeneracy does not affect the existence of SIT single pulses, though the shape of the envelope is slightly altered (Rhodes, Szoke, and Javan 1968). We assume that the same minimal changes would be required for the infinitely long solutions to the Bloch equations which model a steady state maser.

One simple way of including the effects of pumping in the density matrix equations is to alter equation (6) to read:

$$\dot{Z} = -pEY + \lambda(1 - Z) \tag{6a}$$

where $p = \frac{\mu}{\hbar}$. This form of pumping just increases the population density in the ground state, and therefore in the absence of a field E, Z would relax to total inversion. We will assume that the only statistical effect that the pump mechanism has on the molecules can be accounted for by including the reciprocal pump rate $\lambda^{-1}$ in $T_2$, which gives for $T_2$ a minimum of about $10^1$ sec, as we mentioned before.

One of the great advantages of using the slowly varying envelope approximation in addition to examining only the unchirped (constant phase) solutions for the steady state system is that Maxwell's equation reduces to a single linear first order differential equation, viz.

$$\frac{c}{n}\frac{\partial E(z,t)}{\partial z} + \frac{\partial E(z,t)}{\partial t} = \frac{2\pi N p \hbar \omega_o}{n^2} \int_{-\infty}^{\infty} Y(z,t,\delta)\, g(\delta)\, d\delta \quad . \tag{7}$$

The meaning and implications of these various approximations, as well as the solutions for the amplified wave trains to equations (4), (5), (6a), and (7) can be found in a thesis by the author (Rosen 1974b), and the references therein. Suffice it to point out here that the solutions found there correspond to slowly amplifying the steady state solutions found earlier by Crisp (1969) and Eberly (1969), such that $\lambda\tau \ll 1$, where again $\tau = 2/pE_o$. The solutions are:

| Class 1 | | Class 2 | |
|---|---|---|---|
| $X = \dfrac{2\delta^2\tau\, \mathrm{dn}\,(u;k)}{[(k^2 - \delta^2\tau^2)^2 + 4\delta^2\tau^2]^{1/2}}$ | (8) | $\dfrac{2\delta\tau k\, \mathrm{cn}\,(u;k)}{[(1 - \delta^2\tau^2)^2 + 4\delta^2\tau^2 k^2]^{1/2}}$ | (8′) |
| $Y = \dfrac{2k^2\, \mathrm{sn}\,(u;k)}{[(k^2 - \delta^2\tau^2)^2 + 4\delta^2\tau^2]^{1/2}}$ | (9) | $\dfrac{2k\, \mathrm{sn}\,(u;k)\, \mathrm{dn}\,(u;k)}{[(1 - \delta^2\tau^2)^2 + 4\delta^2\tau^2 k^2]^{1/2}}$ | (9′) |
| $Z = \dfrac{-(2 - k^2 + \delta^2\tau^2) + 2\,\mathrm{dn}^2\,(u;k)}{[(k^2 - \delta^2\tau^2)^2 + 4\delta^2\tau^2]^{1/2}}$ | (10) | $\dfrac{-(1 + \delta^2\tau^2) + 2\,\mathrm{dn}^2\,(u;k)}{[(1 - \delta^2\tau^2)^2 + 4\delta^2\tau^2 k^2]^{1/2}}$ | (10′) |
| $E = \dfrac{2}{p\tau}\, \mathrm{dn}\,(u;k)$ | (11) | $\dfrac{2k}{p\tau}\, \mathrm{cn}\,(u;k)$ | (11′) |

with

$$\tau = \frac{\tau'}{K(z)} \tag{12}$$

where $u = (t - z/v)/\tau$, k is the elliptic function amplitude, K is the complete elliptic function of the first kind, and $\tau'$ is a constant. The velocity v approaches c/n as amplification occurs, and the energy density of the radiation

$$u(z) \simeq \frac{n^2 E_o^2\, e^{A(z-z_o)}}{8k_o^2 K} \qquad \left(A = \frac{4\lambda T_2^* N p^2 h\omega_o \tau'}{cn}\right) \tag{13}$$

when $\tau >> T_2^*$, as is the case in astrophysical masers. Since $K\tau$ is constant, the distance between maxima of E remain constant, as the oscillations of E gradually deepen during amplification. Once $k \approx 1$ a series of hyperbolic secant-like pulses form which continue to narrow in width proportional to $K(z)^{-1}$. The ratio of the pulse width to the spacing between pulses also decreases in this manner.

Those solutions also assume that when $\tau << T_2$, that $T_2$ can be effectively set equal to infinity, just as $T_1$ was. For single pulse SIT, when $T_2$ is finite the "pulse area" changes only proportional to the factor $(1 - \tau/T_2)$, to first order in $\tau/T_2$, which for $\tau << T_2$ is close to 1. (McCall and Hahn 1969). The key point is that when $T_2$ is not infinity, Self-Induced Transparency still occurs. McCall and Hahn have further claimed that for a train of SIT pulses $T_2$ "still is required to be much longer than an individual pulse width [$\tau$], but no restriction need necessarily be placed upon the time span of the total train compared with $T_2$", to preserve the basic SIT phenomena. Diels and Hahn (1973) seem to support this position by their finding that only higher order terms implying much greater distances than $T_2$ can cause steady-state pulses or continuous carrier waves to deteriorate. In other words, it is only the amount of dephasing that occurs during a single pulse that could destroy SIT in a pulse train. Any dephased dipoles are shortly brought back into phase with the wave by the same nonlinear interaction that induces SIT-type coherence in the first place, probably in a time of the order of an absorption length divided by the speed of light. Of course, $T_2$ being finite does produce a slow energy loss but this loss is more than made up for by the pumping (see Appendix B).

b) The Relevance of the Solutions for Astrophysical Masers

On the basis of these assumptions, which are verified numerically by Friedberg and Rosen (1976), we postulate that the amplified SIT wave trains previously derived by the author qualitatively model the equilibrium solutions for light with arbitrary initial statistics and spectrum incident on an inhomogeneously broadened and pumped medium when $\tau << T_2$, and $W \geq \Gamma$. Such somewhat randomly fluctuating and thus only partially coherent wave trains would then be present in those outer portions of astrophysical OH and $H_2O$ clouds that contained sufficiently strong electric fields, and would represent the one dimensional solutions for the amplification of radiation in those clouds. (The minor changes required for the spherical solutions will be discussed later). This assumes, of course, that the time for attaining this equilibrium state between radiation and molecules is much less than the time the maser beam takes to traverse that portion of the cloud with strong enough fields to allow for coherence to develop. Again, this crucial assumption is justified by the numerical calculations of Friedberg and Rosen (1976).

Before describing the three basic stages of maser amplification it is useful to review some facts relating to maser geometry.

c) General Geometrical Constraints on the Solutions

From typical but powerful OH sources like W3, Very Long Baseline Interferometric (VLBI) measurements have given us some clues as to the size of the class I OH masers, many of which are widely scattered about a region of only a few seconds of arc, presumably near an HII region (Moran, et al. 1968). Estimates for the apparent sizes average about $5 \times 10^{14}$ cm. for W3-OH (compared to $\sim 5 \times 10^{15}$ cm. for W49-OH and $3 \times 10^{13}$ cm. for W49-$H_2O$), with individual emission points having quite different line center stream velocities separated by about $10^{16}$ cm. (Litvak 1971). Even those values for the interferometric sizes may be suspect, however, since it has been suggested that the linear sizes of OH masers which tend to be about two orders of magnitude greater than $H_2O$ masers (almost all of which remain unresolved) may be accounted for by the relatively greater scattering of the OH maser radiation by inhomogeneities of the interstellar electron plasma (proportional to $\lambda^{-2}$). Indeed, in W49N, "the only case where definite size measurements are available for both the OH and $H_2O$ sources, the ratio of the smallest OH feature (0.05") (Moran 1968) to that of the smallest $H_2O$ feature

(0.0003") (Burke et al. 1972) is nearly equal to the radio of the squares of the wavelengths." (Moran et al. 1973). However, others have argued that interstellar scattering is not an important determinant of apparent maser size, e.g. Boyd and Werner (1972).

Litvak, Goldreich, Lang and Bender and others have proposed that the apparent size of the masing regions as derived from interferometric measurements may not represent the true size of the masing clouds even if interstellar scattering is not important. These measurements may only reflect amplified radiation from a unsaturated hot spherical core or condensation which is further amplified in a fairly spherically uniform way by a surrounding saturated cloud which is much bigger than the core radius (20 to 100 times) (Litvak 1971; Goldreich and Keeley 1972; Lang and Bender 1973). VLBI measurements would then be measurements roughly of the core size, depending on its detailed structure. Most of the radiation coming from the cloud, then, would seem to come from a solid angle subtended by the core, since exponential amplification would only occur when a ray passes through this unsaturated region. Thus the half-power solid angle seen in VLBI measurement would result from all rays that passed close to a diameter of the hot core, since a small difference in distance during exponential amplification leads to large differences in gain. Radiation to be amplified may come from spontaneous emission ($T_B \sim 1$ °K) in the core or the 3°K background, but given the low equivalent temperature for these, the radiation will likely originate for class I sources, especially, in a neighboring HII region, or from the thermal radiation of a hot condensation ($T_K \gtrsim 10^3$ °K) within the core. The core can remain unsaturated given fairly large field intensities within it since the pump rate is so high that the rate of stimulated emission could remain less than the pump rate.

Amplification of the intensity in the saturated outer portion of the cloud is, of course, linear in distance of propagation according to this view, which is based on Litvak's theory of radiative transport discussed in Section II. Goldreich and Keeley (1972) have also found that when the pump rate of the core drops below a critical value, then the entire maser is saturated, and at this point the apparent angular size is at a minimum (Litvak 1973b). Of course, for the whole maser to be saturated it must be amplifying spontaneous emission. Litvak (1973a) has calculated that for parameters appropriate for class I masers, the real (total) size of the masers is about 35 times its minimum apparent size. Of course, this phenomenon of a cloud having a much larger real than apparent size basically reflects the spherical geometry chosen.

Offhand it is difficult to see how spherically shaped pumped regions could arise unless the hot core was itself doing the pumping. Goldreich and Keely (1972) have pointed out two reasons why astrophysical masers may be more like tubes than spheres. The first is that because of the large differences in velocity among individual neighboring emission features relative to the width of these features, long amplification paths may be limited to fairly narrow tubular volumes parallel to the stream velocity of the gas. The second is that, in general, maser pump mechanisms are more efficient when the optical depth for the far-IR resonance radiation is not too great for a dimension perpendicular to the maser axis. In the tubular case, VLBI measurements yield roughly the cross-sectional area. A tubular maser may amplify the 3°K background or, more probably, the black body radiation from clouds of gas behind the tube. Such initial brightness temperatures may be $\sim 10^2 - 10^3$ °K.

On the other hand, if pumped regions were spherical shells surrounding stars or other objects then, of course, one would expect maximum amplification to occur in directions tangential to the shells, and the masers would be more like a flat plate radiating out its edge. But this geometry, too would depend on what the source of amplified radiation was. It seems that the only important aspect that all geometries for clouds have in common is that there is an inner unsaturated region (given sufficient pump rates) and an outer saturated region, such that radiation strikes molecules within the outer regions of the cloud predominantly from a solid angle $\Omega_m$ approximately equal to the square of the ratio of the core radius ($R_o$) (or depth of the unsaturated region), to the distance of these molecules from the peak of the pump rate ($R_m$), i.e. $\Omega_m \cong R_o^2/R_m^2$. Any possible cloud geometry probably implies, then, that the apparent brightness temperatures ($T_B(R_o)$) extrapolated back from the brightness temperatures observed on earth do not reflect the true electric field strengths at the core radius $R_o$ measured by the interferometer, but rather the brightness temperature ($T_B(R')$) at the outer radius $R'$. This means that $T_B(R') \approx T_B(R_o)\,\Omega_m$. This fact somewhat lowers the pump rates required to account for the high brightness temperatures seen, since there is more space in which amplification can occur, though most of the gain probably still is accounted for by the unsaturated core. (Hereafter, the use of the word "core" does not connote spherical geometry.) Thus, when considering our model for astrophysical masers we will not be specifically concerned with the actual geometry or the source of the incoming radiation. Since the first stage in amplification in any model that has greater than critical pump rates involves unsaturated exponential amplification, the different possible temperatures for the incoming radiation are not at all crucial. The different geometries may crucially affect the rate of change over time of a maser's output intensity, but we will not be concerned with the details of that problem here. Even for a tubular model we assume that the radiation, being confined to a finite but small beam angle, would not be a plane wave and the intensity would therefore fall off proportional to $R^{-2}$ after leaving the masing cloud, just as if we were considering a spherical model.

## IV. Model for a Coherent Maser

### a) Stage One: Un-Saturated Amplification

We assume with Litvak (1971) that the maser amplifies radiation with a fairly high brightness temperature ($T_B \approx 10^2$ °K) from a background source radiating like a black-body, and which therefore has Gaussian statistics. Using Goldreich and Keeley's figures for OH masers (we will discuss the changes that $H_2O$ masers require later), we will assume that the kinetic temperature of the gas is approximately $10^2$ °K, and thus the full Doppler width at half-power $\delta\nu_D$ of the maser radiation with $\nu_o \simeq 1.7 \times 10^9$ Hz. is $\delta\nu_D \simeq 1.7 \times 10^3$ Hz., which corresponds to a velocity shift of $v \cong .53$ km/sec. This stage involving unsaturated amplification of the Gaussian radiation, and the corresponding narrowing of the linewidth has been adequately described and summarized previously, e.g. by Litvak (1970b) and Sullivan (1973). Beer's law ($I(z) = I_o e^{\alpha' z}$) applies here where:

$$\alpha' = \frac{4\pi^{3/2} \Delta N p^2 \hbar\omega_o T_2^*}{nc\,(2F+1)} \tag{14}$$

Since the pump rate $\lambda$ is roughly equal to the rate of relaxation via collisions across the two molecular levels, $\Delta N$ is probably $\geqslant 10^{-2}$ N, where N is the density of active molecules, but we will use $\Delta N = 10^{-2}$ N here to be conservative and to conform to the assumptions of Goldreich and Keeley, and Litvak. Substituting we find that $\alpha' \approx \frac{6 \times 10^{-12}}{(2F+1)}$ cm.$^{-1}$, i.e. Beer's length $\alpha'^{-1} \approx 6.5 \times 10^{11}$ cm. for F = 2.

The basic physics of these masers is embodied when only treating the molecules as two-level systems, since for OH $\delta\nu_D$ is much less than the hyperfine splitting of the levels involved. Thus the effects of neighboring hyperfine levels on masing is taken into account by the non-resonant effects of these levels included in n, the index of refraction, along with the effects of all the other molecular levels.

The condition for saturation is that the stimulated rate of emission from the total field $E_t$,

$$W \simeq \frac{\pi p^2 E_t^2}{\delta\nu} \tag{15}$$

is approximately equal to the pump rate, so that the population inversion $\Delta N$ becomes altered from its unsaturated value. In terms of the brightness temperature of the radiation

$$W \simeq \frac{kT_B A \Omega_m}{4\pi\hbar\omega_o} \tag{16}$$

Thus for saturation

$$T_B\Omega_m = \frac{4\pi\hbar\omega_o\lambda}{kA} \approx 1.4 \times 10^8 \,{}^\circ\mathrm{K\ ster.} \tag{17}$$

Though, as we have discussed previously, $\Omega_m$ is somewhat uncertain at a given distance from the core, but at the onset of saturation $\Omega_m$ must be $\gtrsim 10^{-1}$, for amplification of W by about $10^6$. Thus the local $T_B$ for saturation is $\approx 10^9\,{}^\circ$K. For other values of $\Omega_m$ we can adjust $T_B$ at saturation accordingly. To amplify the background radiation to this point $E_t^2$ grows by a factor of $10^6$ divided by the fractional change in linewidth due to unsaturated narrowing. For amplification of $10^6$ we require $\alpha' z = \ln(10^6) \approx 14$. Therefore the distance ($z_s$) in which unsaturated amplification occurs is about $z_s \approx 10^{13}$ cm.

Since a small change in the distance will cause a large change in the amplification, this figure must be fairly accurate. It is also important to note that at this point $\delta\nu_D$ has been narrowed by $(\alpha' z/\ln 2)^{1/2} \approx 4.5$, thus $\delta\nu \approx 3.8 \times 10^2$ Hz. Thus $E_t^2$ has been multiplied by $\sim 2 \times 10^5$ and $E_t$ has attained a value from expression (15) of

$$E_t \simeq \sqrt{\frac{\lambda \delta\nu}{\pi p^2}} \approx 10^{-9} \text{ e.s.u.} \tag{18}$$

Noting that our condition for the possibility of coherence becoming established in addition to $W \geq \Gamma$ was $pE >> T_2^{-1}$, if $T_2 \approx 10^{-1}$ sec. then this occurs when $E >> 10^{-10}$ e.s.u. This latter criteria always becomes satisfied for weaker fields than imply saturation in the broadband case, as can be seen from equation (15). If we believe that the true size of a hot core or tubular cross section is $\sim 3 \times 10^{13}$ cm. in the OH case, too, as indicated from $H_2O$ maser measurements, then perhaps the values we have used for $\Delta N$ or $T_2^*$ are too big, or $\lambda$ is too small given the unsaturated amplification distance we have derived. In fact, if we also assume that one pump photon produces one maser photon then for a maximum pump rate $\lambda \approx 10^{-1}$ sec$^{-1}$, we require a region of space at least $5 \times 10^{14}$ cm. in radius for the total maser size. This is, of course, consistent with the core size being a factor of 10 smaller. We might believe that the $H_2O$ VLBI measurement is fairly accurate for the hot core size of OH, because at the $H_2O$ wavelengths little interstellar scattering should occur even if scattering is significant for OH masers. If we believe that the VLBI sizes measure more than the core, then our parameter values may even err in the opposite direction. At any rate, the $\Omega \approx 10^{-1}$ figure at $z_s$ is not too far off and is surely not much too big given the VLBI sizes compared to $z_s$ above.

b) Stage Two: The Traditional View

According to non-coherent maser models such as that of Litvak, the second stage of amplification involves approximately linear growth of the intensity, and saturated rated broadening of the linewidth back out to the Doppler width divided by $\sqrt{2}$ (or perhaps $\sqrt{3}$ ), to take account of spherical spreading, within roughly a distance of $10\ z_s$ or $7 \times 10^{13}$ cm., from the point of saturation ($z_s$) Litvak 1973b). This can be seen from the formula for saturated line broadening:

$$\frac{\delta\nu}{\delta\nu_s} \simeq \{a'(z-z_s) - \ln[1+(z-z_s)] + 2\}^{1/2} (a'z)^{-1/2} \qquad (19)$$

where $\delta\nu_s$ is the saturated linewidth ($\delta\nu_D/\sqrt{2}$ here) (Litvak 1970b). Of course, by this time $T_B$ has only increased to about $10^{12}$ °K for $\Omega_m \approx 10^{-2}$, though this depends critically on $\Omega_m$, as we have already discussed. Thus if $\Omega_m \geq 10^{-2}$ at $T_B = 10^{12}$ °K (which seems likely given the VLBI sizes relative to $10\ z_s$), the linewidths of the brightest lines in this model should all have broadened out to anywhere from $\delta\nu_D$ to $\delta\nu_D/\sqrt{3}$, for $R_m \geq 10^{14}$ cm. In this view, after $10^{14}$ cm., no further broadening should occur. These numbers seem consistent with Litvak's own calculation that the size of strong OH masers is $\sim 35\ z_s$ for $T_B \sim 5 \times 10^{13}$ °K and $\Omega_m \sim 10^{-3}$, and thus all lines with $10^{12}$ °K $\leq T_B \leq 5 \times 10^{13}$ °K should have roughly the same width.

c) Stage Two: The New Model -- The Onset of Coherence

On the other hand, we construct a model of an astrophysical maser that has a coherent radiative output with non-Gaussian statistics of the type derived by Rosen

(1974b). We will now supplement the argument given in Section II as to how this change in the character of the radiation takes place, with arguments from other authors consistent with it. At the end of stage one the field amplitude would look like random pulses of both positive and negative amplitude with an average width of the pulses characterized by $T_2^*$, The Doppler width, since $T_2^*$ is also approximately the autocorrelation time. If we apply the criteria of the area theorem to the radiation at this point we see that $A \simeq pET_2^* \approx 10^{-3}$, that is, the average pulse area (A) is much less than $\pi$ (see McCall and Hahn (1969) for an explanation of the significance of "pulse area"). Coherent pulses, probably of Crisp's (1969) class 2 type for reasons of symmetry, would readily form from noise these pulses if the average pulse length increased (spectrum narrowed from Doppler width), as further non-coherent saturated amplification occurred. As we mentioned earlier, this happens once saturation sets in, since now the field would induce oscillations in the microscopic dipole moments at a frequency W, which would then radiate coherently back into the field at this same rate, tending to lengthen pulses in time since $W^{-1} >> T_2^*$. Random pulses with larger initial areas would tend to absorb smaller pulses into them. This type of behavior has been seen in the laboratory with self-pulsing lasers by Kryukov and Letokov (1972). Miller and Szoke (1973) have also put forth a similar argument to the same effect, though some of their conclusions differ from these here quantitatively. In addition, Icsevgi and Lamb (1969) have done some numerical calculations which show that single small area pulses in an amplifier tend to broaden rapidly in time until their areas are $\approx \pi$, before they narrow again. This occurs within about ten Beer lengths. (Of course, saturation will increase the effective Beer length by reducing $\Delta N$). Diels and Hahn (1973) show that if a coherent pulse ($\tau << T_2$) is started off-resonance in an amplifier then frequency pulling towards resonance occurs. This makes it plausible that the frequency range of

Gaussian radiation narrows (broadening the pulse in time) once saturation occurs and the condition for coherence is met, implying that non-linear forces will dominate. Indeed, initial computer calculations of Lamb (1974) and Hopf (1974) indicate that when Gaussian noise is amplified the frequency bandwidth narrows to about pE, and Hopf reports that a train of pulses develops also within about 10 Beer lengths similar in most respects to the ones described in Section III, though somewhat irregular in pulse height, spacing and width. These findings have all been confirmed in one-dimension by the computer calculations mentioned earlier due to Friedberg and Rosen (1976).

From this perspective, stage two consists of a slowing down of exponential growth for the electric field due to saturation over a distance approximately equivalent to that over which unsaturated amplification occurred, i.e. $\sim 10^{13}$ cm. However, as the radiation becomes more coherent the intensity again grows exponentially as we shall describe in the next stage, though at a different rate. At the end of this stage the field will be $\geq 10^{-8}$ e.s.u and may have established a somewhat randomized form of the cn(u;k) envelope. Simultaneously, the spectrum could have narrowed as much as to $\delta\nu \approx \tau^{-1} \approx 15$, viz. by a factor of about 20, for the values of the parameters we are using here, though no lines this narrow are observed. Therefore, the brightness temperature $T_B$ reaches $10^{12}$ °K at a distance of roughly a few times $10^{13}$ cm., since narrowing the spectrum increases the brightness temperature even when $E_t$ remains constant. Of course, interstellar scintillations, turbulence, other scattering effects, and the many uncertainties in the values of key parameters may shift this linewidth at which coherence is established by an order of magnitude or more higher to account for the fact that no lines this narrow are seen.

## d) The Structure of the $H_2O$ Maser

We must also briefly consider the changes that occur in the rough proportions and numbers that we have cited using the OH data for parameters, when we consider water vapor masers. Let $\lambda = 10^{-1}\ s^{-1}$, $N_{H_2} = 10^9\ cm^{-3}$, $N_{H_2O} = 3 \times 10^4\ cm^{-3}$, and $T_K = 10^3\ {}^\circ K$ (Goldreich and Keeley 1972). For $H_2O$, then, $\alpha'^{-1} \approx 5 \times 10^{12}$ cm for $\Delta N = 10^{-2}\ N_{H_2O}$, which is at least a factor of 10 smaller than implied by $z_s = 3 \times 10^{15}$ cm. obtained by Sullivan (1973), for a completely saturated maser of $T_B \sim 10^{15}\ {}^\circ K$. If we take $3 \times 10^{13}$ cm. from the VLBI measurements as roughly the hot spot cross-section, and $z_s \leqslant 20\ \alpha^{-1} \approx 10^{14}$ cm. as the length of the hot spot, then $\Omega_m \approx 10^{-1}$ near the cloud periphery of an unsaturated maser again, and $\Omega_m \approx 10^{-3}$ for a fully saturated maser. Thus these masers may be very similar in structure to OH masers, though it is likely that higher pump rates and densities may allow for smaller total maser sizes of $\sim 3 \times 10^{14}$ cm for $\lambda \sim 10^{-1}\ sec^{-1}$ and thus relatively smaller outer saturated regions with respect to core sizes $z_s$. The latter may account for the greater observed intensity fluctuations over time in $H_2O$ masers than in OH masers.

## e) Stage Three: Coherent Amplification

Once coherence has been established then the results of the author's earlier paper (Rosen 1974b) for amplification with $\tau >> T_2^*$ provide a guide to what happens to an astrophysical maser. In this case the energy density U grows $\propto e^{Az}$, and $W_{max} \cong \pi p E_{max}$ when $\delta\nu \simeq \tau^{-1} = pE_{max}$, during the pulse. However, since the average pulse width decreases with respect to the average pulse period during amplification then over a complete period the average rate of stimulated emission ($W_{av}$) is:

$$W_{av} \cong \frac{\pi (p\, E_{rms})^2}{\delta\nu} = \frac{\pi (p\, E_{max})^2}{\delta\nu} \;\frac{\tau}{K\tau} = \frac{4\pi}{\tau'} \qquad (20)$$

i.e., a constant, and $(T_B \Omega_m)$ seems to remain constant though U increases. Thus, $\delta\nu$ is proportional to $E^2_{rms}$ during coherent amplification.

If $\tau'$ is chosen to be about $10^{-3}$ corresponding to the narrowest OH lines observed, then the length $A^{-1}$ is about $10^{14}$ cm., so all necessary further amplification could occur within a few times $10^{14}$ cm, which is consistent with our earlier total size estimate for $\lambda \sim 10^{-1}$ sec$^{-1}$. Exponential coherent amplification over distances of this order implies that there would be less of a difference between the apparent (VLBI) and the real size of the maser than in Litvak's (1973a) model where exponential amplification only occurred in the unsaturated core. Actually, of course, amplified solutions should be obtained in the case of spherically symmetric geometry to account for the spreading of the radiation through a solid angle. However, solving in this case we find that $K \propto e^{A(r-r_o)}/r^2$ instead of $K \propto e^{Az}$, where r is the radial coordinate. U(r) is still proportion to K(r). This implies that the exponential factor will quickly dominate the $R^{-2}$ factor, for physically reasonable $r_o$ (the radius at saturation), and the distance estimates of the previous paragraph will still be approximately correct, though they will tend to be too short.

f) Past Argument Opposing Coherence

At this point we must discuss the validity of previously published arguments which deny the possibility that pulses could develop in an astrophysical maser if the maser radiation is not confined to very small solid angles, simulating plane wave behavior. Instead the maser output is the result of the interaction of plane waves propagating at large angles with respect to each other in the core, as would be the case in a spherical model, and then propagating through the saturated region. Specifically, Goldreich, Keeley, and Kwan (1973) have argued that three-dimensional input from independent sources of non-coherent radiation precludes the development of coherent pulses of any sort in an amplifying medium, and they have been supported by Litvak (1973b). They correctly point out that to have a wave with non-stationary (non-Gaussian) statistics, definite phase relations must be maintained among the Fourier components of the field. This is, of course, the reason why Crisp's solution propagate with no distortion at a constant group velocity through a two-level medium. However, then, they proceed to claim that "a pulse may propagate between two points separated by many wavelengths only if the phase velocity at different frequencies are almost the same". This contradicts their earlier statement that the phase velocities at different frequencies (the same as velocities of Fourier components, we assume) could be different as long as phase relations were maintained.

The remainder of the argument against pulses states that since there is a relative phase shift ($\delta = 2\pi d \theta^2 / \lambda$) along the direction of propagation of one plane wave, with respect to the phase of another plane wave propagating at an angle ($\theta$) with respect to the first after a distance d, then when $\delta \geq 2\pi$ the coherence will be destroyed. However, this argument would seem to apply equally to show that an interference pattern in a linear medium could not travel greater than a distance $d \sim \lambda / \theta^2$, which is certainly not true. Furthermore, the interference pattern propagates in the direction where there is no relative phase shift between the waves, which obviously is not along the direction of either plane wave.

At any rate, the argument is useless against the possibility of coherence for other reasons entirely. Coherence is not a linear phenomenon that can be treated by simple considerations of phase shift, diffraction, or interference of multi-directional waves. It is the result of complicated fairly local non-linear interactions between the field and the medium in which it propagates, which often results in steady-state propagation given interaction distances approximately 10 amplification lengths or more. The interaction causes correlations in the phase of different Fourier frequency $(\omega)$ and wave vector $(\vec{k})$ components to develop (Mollow 1972), thus destroying stationary statistics. As we have already pointed out, such correlations define what coherence is. Since Goldreich (1974) admits that coherent radiation would probably develop in an amplifying medium from noise in one dimension, it is not clear why this could not happen for some other symmetry. It is mostly the interaction with the medium of converging waves within the last 10 $z_s$, or so of the maser's edge that determines the output. The spherical solutions to the density matrix and Maxwell's equations mentioned earlier describe a pattern of coherent pulses that could be decomposed into multi-directional plane waves. Thus these spherical solutions could qualitatively model radiation emitted locally from a sphere or region with similar local geometry where coherence would be established beyond a certain critical radius (about 25 Beer lengths), and where the input to this coherent region would come from waves propagating in a continuum of different directions. Physically the coherent waves could be somewhat irregular in both the radial and transverse direction as Hopf's and Friedberg and Rosen's one dimensional computer calculations were, and still on the average have the basic properties described by the amplified SIT wave trains described here.

In a recent paper Litvak (1973a) has also argued that emitting regions of coherent radiation would be limited in size to $10^6$ or $10^7$ cm., since these were the coherence distances for light of frequency spread of $10^3$ or $10^4$ $sec^{-1}$. Therefore any astrophysical maser would have to be composed of many such regions masing independently of each other and by the central limit theorem the radiation from some regions shining on others would yield radiation with Gaussian statistics. However, these coherence distances only result from a linear optics argument when Gaussian statistics are assumed, by dividing c by $\delta\nu$, and are not a general result for radiation interacting with matter.

## V. The Evidential Support for the Coherent Model

In this section we must consider the quality of existing observations that have been made of astrophysical maser emission, and its relevance for deciding whether the coherent model outlined in the previous section can better describe the data than the traditional view. Again we will basically limit our detailed discussion to class I OH masers, though relevant facts from other classes will be considered when appropriate. The observational differences between $H_2O$ and OH sources will be analyzed in light of the predictions of Section IV.

### a) Description of the Data Available

It is fortunate that since we expect our theory to be most relevant to class I OH sources, the best data, i.e. the data having the highest frequency resolution, exists for precisely some of these sources. Rönnäng (1972) obtained plausible Gaussian fits to almost all the spectral lines he observed for at least six of the major class I emitters at a resolution of 250 Hz. Most previous data had been obtained at a resolution of 1 kHz or 3 kHz, which implies that since many features overlap in a source, and since linewidths average 1 kHz, one could never be sure whether a single or multiple spectral line was represented by a spike (Wilson et. al. 1970; Robinson et. al. 1970). Clearly, it will be vital to compare predictions of linewidths and intensity between the coherent and traditional models. Therefore, we can be the most confident using Rönnäng's data for this purpose. The third major feature of these class I lines, viz. their very high degree of circular polarization, has been clearly established even by the lower resolution observations, and thus the polarization derived from these observations may underestimate the strength of this phenomenon. Unresolved oppositely polarized lines would tend to appear as unpolarized, if the lines were comparable in intensity. Unfortunately, Rönnäng did not collect polarization data. Regarding the presence of data that might reflect on whether or not astrophysical maser radiation have non-Gaussian statistics, only one set of measurements have been made, but as we shall see, there are many factors which complicate the interpretation of these measurements.

The relationship between linewidth and intensity best allows us to test whether any evidence exists for coherent radiation in astrophysical masers, given the existing data. Several theories to explain polarization have been offered, and though there are problems with them, the issue is not as clear cut in that area (Goldreich, Keeley, and Kwan 1973a, b; Hall and ter Haar 1973). Briefly, again, the problem with the traditional view arises because circumstances of pump mechanism, level excitation required, or location would tend to imply that the thermal temperature ($T_K$) of the class I OH masing gas would be $\gtrsim 100\,^\circ K$, and perhaps considerably higher. Assuming $T_K = 200\,^\circ K$ for example, unsaturated amplification would perhaps narrow the spectral lines to an effective temperature width ($T_e$) of $15\,^\circ K$, and indeed many lines with such a width have been observed, though lines as narrow as $7\,^\circ K$ have also been observed. But the lines with $T_e \approx 15\,^\circ K$ are quite intense enough to be saturated. Their high degree of circular polarization (often greater than 90%) supports this view as a requirement of all existing polarization theories, as well as the fact that the different OH hyperfine states are similar in intensity in spite of quite different line strenths. (Goldreich and Kwan 1974). Furthermore, as we have demonstrated theoretically in Section IV, the values of parameters used (especially the pump rate) imply that such strong class I line ($T_B \sim 10^{12}\,^\circ K$) can only be achieved in saturated masers. This prediction is itself based on the traditional maser model. If the lines are saturated, though, we have already pointed out that their widths must be equal to at least $T_k/2$ or $T_K/3$ for spherical symmetry, if not $T_K$ for cylindrical symmetry. This would imply that $T_k$ is usually $15(3) = 45\,^\circ K$, or less, down to as low as $15\,^\circ K$, and is implausible.

If gas temperatures were this low ($T_k \leqslant 45°K$) it has been pointed out that the excited OH rotational states would not be highly excited enough to radiate with their observed intensities. Furthermore, Sullivan (1973) finds $T_k \approx 500°K$ a good estimate for the $H_2O$ masing clouds in the same vicinity as the class I OH masers, which if true for OH also, would imply that unsaturated gain narrowing could never explain lines as narrow as $T_e \sim 10°K$.

b) Evidence for the Coherent Model

Now we turn to the advantages of our coherent model for astrophysical masers for explaining the available data, especially Rönnäng's data. The primary question to be asked is can we pick out any spectral lines, or even better a set of lines from the same source, which seem to be coherent. Fortunately, we can, as is seen in Table I which lists the right and left circularly (RC and LC) polarized emission lines from W3 at 1665 MHz. The best example is provided by the five of the six brightest LC lines above the dotted line. Column 3 shows the value of the integrated flux received on earth ($F_e$), which is proportional to $E_{rms}^2$ divided by the linewidth $\delta\nu$. The reason this quantity is of interest is because our coherence theory predicts that the flux (F(R)) of a spherical maser at any radius R, which is proportional to $E_{rms}^2$ (R) divided by $\delta\nu$ at that point, would be a constant independent of F. This was mentioned previously in Section IV, and the expression for F is:

$$F(R) = \frac{c}{4\pi} E_{rms}^2(R) \cong \frac{c}{4\pi} \frac{E_{max}^2 \tau}{2K\tau} = \frac{c}{2\pi p^2 \tau'} \left(\frac{1}{\tau}\right) \tag{21}$$

where $1/\tau = \delta\nu$, and $\tau' = K_o \tau_o$. After leaving the masing cloud the radiation continues to spread out through the solid angle whose vertex is the symmetry center of the maser, so that the flux received on earth $F_e = (R/R_e)^2 F(R)$, where $R_e$ is the distance from the earth to the center of the maser. Thus $F_e$ divided by $\delta\nu$ will be just:

**TABLE I: 250 Hz OH Maser Source Spectral Data** (from Rönnäng, 1972)

| FLUX ($10^{-22}$ w/m$^2$) | $\delta\nu$ (km/sec) | FLUX / $\delta\nu$ | $T_e$(°K) | Line Center Vel. - (km/sec) |
|---|---|---|---|---|
| W3 - 1665 - LC | | | | |
| 44.2 | .592 | 75 | 130 | -46.31 |
| 22.6 | .409 | 55 | 62 | -45.54 |
| 14.2 | .323 | 44 | 39 | -45.02 |
| 8.4 | .246 | 34 | 22 | -46.62 |
| 7.6 | .224 | 34 | 19 | -45.31 |
| 11.6 (out of order) | .562 | 21 | 117 | -44.47 |
| 6.3 | .263 | 24 | 26 | -49.09 |
| 2.3 | .289 | 8.0 | 31 | -47.40 |
| 2.0 | .494 | 4.1 | 91 | -48.66 |
| 1.4 | 1.6 | .88 | 962 | -47.67 |
| 1.1 | .150 | 7.4 | 8 | -44.54 |
| .8 | .268 | 3.0 | 27 | -43.73 |
| W3 - 1665 - RC | | | | |
| 53.7 | .332 | 162 | 41 | -45.05 |
| 19.0 | .289 | 66 | 31 | -43.77 |
| 15.5 | .263 | 59 | 26 | -45.30 |
| 9.6 | .511 | 19 | 97 | -41.25 |
| 8.5 | .502 | 17 | 94 | -44.31 |
| 5.8 | .357 | 16 | 47 | -41.72 |
| 4.6 | .259 | 18 | 25 | -43.03 |
| 3.7 | .558 | 6.6 | 115 | -42.24 |
| 2.3 | .268 | 8.6 | 27 | -42.70 |
| 1.2 | .242 | 5.0 | 22 | -49.05 |

$$F_e/\delta\nu = \frac{c}{2\pi p^2 \tau'} (R_o/R_e)^2 \tag{22}$$

where $R_o$ is the outermost radius at which amplification occurs.

If there is a fairly uniform region of an OH cloud which has several individual maser sources within a small region, we might expect that the masers would have very similar values for their basic parameters (and $\tau'$), and thus very similar structures. Furthermore, if the amplification rates are the same for these masers, then those that appear most intense will have a greater value for $R_o$. Returning to the five W3-1665-LC lines we see that, indeed, they follow the definite pattern of coherent broadening where $\delta\nu$ increases linearly with F. The deviation of the ratio of $F_e/\delta\nu$ from a constant can be explained by the necessity for the sources with the greatest F to have a greater $R_o$. Indeed, if we plot $F(R_o)$ vs. $R_o$ for these lines, we get a good fit (to within 5%) to exponential growth with distance thgough the cloud over distances $\sim 10^{14}$ cm., again assuming the sources are almost identical to each other. This later assumption is plausible given the small spread in center velocities between the lines, and the VLBI measurements that show these sources are closely grouped spatially. Independent support for this interpretation of the radiation comes from noting that when the ratio for $F/\delta\nu$ for the five strong LC lines is projected back through the solid angle to W3, it corresponds to the proper ratio $(c/2\pi p^2\tau')$ for coherent radiation of the type we predict also at values of $R_o \sim 10^{14}$ cm. This is almost exactly the size which Litvak (1971) lists for W3-OH from the VLBI measurements, and which is quite plausible as the outer radius of the maser given our calculations in Section IV. Exponential coherent amplification also occurs over distances of the order $A^{-1}$, and $A^{-1} \approx 10^{14}$ cm. if $\tau'$ is changed from $\sim 10^{-2}$ sec., as was used in Section IV for the estimate of A to $\sim 10^{-3}$ sec. as is required here given the width of the F = 7.6

LC line. Thus our theory is consistent with the observed data, though the observed data alone is not complete enough to conclusively select among the various theories proposed.

The fit to our theory by the five W3-1665-LC lines is especially amazing given the many parameter values that theoretically could be somewhat different for each line. That the five line sequence is remarkable can be seen by looking at the values for lines with lower intensities. Again, from the theory we would expect the values of the ratio in column 3 to be much smaller for these weaker lines. This difference is caused by the narrowing of the lines below the minimum attained by unsaturated narrowing when coherence begins to set in as for the F = 7.6 line. Thus, in general, it is easy to explain the presence of any saturated line observed with $T_e < 40\,^\circ K$ as mostly or fully coherent, but again, it is more significant if neighboring groups of coherent lines can be found. To repeat, the most obvious pattern among neighboring maser sources indicative of coherent lines would be when strong lines are particularly narrow, after which stronger lines become monotonically broader with increasing intensity. Unfortunately, most sources listed in Table I, II, and III do not contain many well measured lines so that clear-cut patterns cannot be perceived, though some other lines do have values for $F/\delta\nu$ roughly the same as for the W3 lines believed to be coherent. Of course, if $R_e$ is greater for the other clouds, such as for W49, than for W3, then the ratio $F/\delta\nu$ need not be as large to signify coherence, though this depends on $R_o$ for that source.

Another group of lines which are probably coherent and which seem to show coherent broadening are the three brightest RC polarized lines in W3 at 1665 MHz (see Table I). In this case the value for $F/\delta\nu$ is even higher than for the LC lines, and this may indicate that the amplification lengths for this sense of polarization are somewhat longer than for the LC radiation. It is also important to note that according to Moran's spatial diagram of

W3, the neighborhood of these three RC lines borders on the five LC lines, and indeed, two of the three center velocities of the three RC lines overlap the center velocities of the LC lines (Moran et al. 1968). Looking at the weaker lines, the RC line with flux (F) of 4.6 and the LC line with F = 6.3 probably represent significant partical coherence, and both $T_e \approx 25\ ^\circ K$. The other weak lines are probably less highly coherent, though since lines this bright are believed to be saturated, most all probably have some degree of coherence, for otherwise they would be much broader. As we described in Section IV, since saturation is the criteria for the possibility of coherence, the onset of coherence would tend to narrow the lines, offsetting the usual saturation broadening. Thus the onset of coherence allows the linewidths to remain near their minimum values attained during unsaturated amplification. It is unfortunate that Rönnäng's data does not show the degree of polarization for the lines, for then another measure of the degree of saturation for the lines could be established, assuming that unsaturated lines would not be polarized.

The source W75B also seems to emit an interesting set of RC lines from this point of view as can be seen in Table II. The values of the ratio $F/\delta\nu$ in column 3 increase almost exactly monotonically with increasing flux, independently of the line center velocity, with the brightest lines clearly being the narrowest. Except for a couple of features, W51 RC and LC also follow this pattern. The sharp jump in $F/\delta\nu$ for W75B RC which begins after one of the F = 3.4 lines may indicate that the onset of a high degree of coherence. Assuming fairly constant parameter values throughout W75B, we reiterate that it is hard to see how the traditional view of Litvak's could explain both increasing intensity and continued narrowing. Similarly, it seems that Goldreich and Kwan's (1974) new hypothesis could not explain the rapid rate of narrowing given the relatively small increase in intensity. From our point of view the most intense W75B RC line has probably only just about reached the fully coherent state, since no coherence broadening is seen. Again the several weaker lines with $T_e \approx 50\,^\circ K$ may represent partially coherent lines which have remained near the point of maximum unsaturated narrowing, and would imply that the kinetic temperature of the gas for this source is $\approx 10^3\,^\circ K$. Similarly, W51 has many lines with $T_e \approx 37\,^\circ K$ implying $T_k \approx 740\,^\circ K$.

One LC line in W49 at 1720 MHz which is believed to be 100% polarized also is the most intense (F = 18.7), and has a high value for $F/\delta\nu$, as seen in Table II. In general, as we have mentioned, most of the other class I intense lines are also believed highly polarized, but it is difficult to match past measurements with their poor resolution to Ronnang's data. W51 has two spatially neighboring RC lines which may show coherent broadening, for the F = 11.1 and 4.4 ones. (see Table III). Remember, since W51 is more than twice the distance of W3, $F/\delta\nu$ could be considerably lower than the W3 values, and still indicate coherence. Similarly the F = 23.3 and F = 5.1 LC lines in W51 and 1665 MHz may represent some degree of coherent broadening, and the F = 2.3. line has the same center velocity as the F = 4.4 RC line. The two brightest lines in W3 at 1667 MHz for both LC and RC polarization may also indicate coherence broadening. (See Table II). The narrow bright W75B LC line at F = 2.9 may also be substantially coherent. (See Table III). At any rate, it is clear from the data that for each source the most narrow lines, where the Gaussian fits can be believed given the magnitude of the residual fluctuation, are often among the most intense. Thus, almost every source Rönnäng has measured seems to contain some evidential support for the existence of coherence.

**TABLE II : OH Maser Spectral Data — Continued** (from Rönnäng, 1972)

| FLUX ($10^{-22}$ $w/m^2$) | $\delta\nu$ (km/sec) | $\frac{FLUX}{\delta\nu}$ | $T_e$ ($^oK$) | Line Center Vel. -(km/sec) |
|---|---|---|---|---|
| W75B - 1665 - RC | | | | |
| 6.2 | .177 | 35 | 12 | 11.83 |
| 5.3 | .211 | 25 | 17 | 11.97 |
| 4.0 | .207 | 19 | 16 | 5.75 |
| 3.4 | .302 | 11 | 34 | 9.31 |
| 3.4 | .549 | 6.2 | 112 | 5.47 |
| 2.1 | .417 | 5.0 | 65 | 12.89 |
| 1.0 | .375 | 2.7 | 52 | 4.91 |
| .8 | .519 | 1.5 | 100 | 6.37 |
| .6 | .357 | 1.7 | 47 | 10.98 |
| .5 | .340 | 1.5 | 43 | 4.37 |
| W49 - 1720 - LC | | | | |
| 18.7 | .425 | 44 | 67 | 14.29 |
| 16.8 | .569 | 30 | 120 | 14.79 |
| .7 | .317 | 2.2 | 37 | 13.28 |
| W3 - 1667 - LC | | | | |
| 4.4 | .387 | 11 | 56 | -44.70 |
| 2.0 | .211 | 9.5 | 17 | -45.05 |
| .6 | .198 | 3.0 | 15 | -44.29 |
| .6 | .408 | 1.5 | 62 | -43.25 |
| .4 | .150 | 2.7 | 8 | -43.94 |
| W3 - 1667 - RC | | | | |
| 1.4 | .220 | 6.4 | 18 | -42.32 |
| .8 | .141 | 5.7 | 7 | -43.11 |
| .7 | .267 | 2.6 | 26 | -42.64 |
| .2 | .181 | 1.1 | 12 | -41.78 |

**TABLE III : OH Maser Spectral Data — Continued** (from Rönnäng, 1972)

| FLUX ($10^{-22}$ w/m$^2$) | $\delta\nu$ (km/sec) | FLUX / $\delta\nu$ | $T_e$ (°K) | Line Center Vel. -(km/sec) |
|---|---|---|---|---|
| W51 - 1665 - RC | | | | |
| 11.1 | .387 | 29 | 56 | 58.82 |
| 8.2 | .545 | 15 | 110 | 58.32 |
| 6.8 | .519 | 13 | 100 | 59.46 |
| 4.8 | .485 | 9.9 | 87 | 57.73 |
| 4.4 | .268 | 16 | 27 | 59.20 |
| 2.1 | .524 | 4.0 | 102 | 60.21 |
| .8 | .323 | 2.5 | 39 | 56.92 |
| W51 - 1665 - LC | | | | |
| 23.3 | .422 | 55 | 66 | 59.22 |
| 8.2 | 1.212? | — | — | 58.56 |
| 5.1 | .280 | 18 | 29 | 57.70 |
| 5.0 | .426 | 12 | 67 | 60.43 |
| 4.3 | .319 | 13 | 38 | 58.75 |
| 3.0 | .315 | 9.5 | 37 | 61.38 |
| 2.3 | .310 | 7.4 | 36 | 56.86 |
| 2.0 | .332 | 6.0 | 41 | 59.71 |
| 1.0 | .298 | 3.4 | 33 | 60.03 |
| .4 | .203 | 2.0 | 15 | 57.24 |
| W75B - 1665 - LC | | | | |
| 3.3 | .323 | 10 | 39 | 4.91 |
| 2.9 | .198 | 15 | 15 | 3.06 |
| 2.3 | .881 | 2.6 | 288 | 5.55 |
| 1.3 | .272 | 4.8 | 27 | 3.43 |
| 1.3 | .323 | 4.0 | 39 | 4.40 |
| .9 | .306 | 2.9 | 35 | 3.92 |
| .7 | .237 | 2.9 | 21 | 5.96 |
| .4 | .101 | 4.0 | 4 ? | 3.23 |

c) Observations of the Maser Statistics and Their Analysis

The only significant attempt to directly measure the statistics of some of the strongest OH emission lines, including some from W3, resulted in no significant deviations from Gaussian statistics (Evans et al. 1972). However, Evans et al. themselves point out many reasons why we might not be able to detect coherent radiation here on earth, even if it existed within the astrophysical masers. The most important of these is the many distorting effects that interstellar scattering by the fluctuations in the electron plasma density may have on the radiation in transit.

Unfortunately, no calculations have yet been done detailing how coherent radiation would be affected by interstellar scintillations. We must remember, however, that once the coherent wave exits from the resonant medium, the stability enforced by the non-linear interaction with the medium no longer exists. Thus the propagation of each Fourier component of the wave can be treated independently, as if the wave had Gaussian statistics. Given sufficient fluctuations in the index of refraction which will now affect the Fourier components differently according to frequency, large phase shifts will be introduced between these components. In addition, Evans, et al. have estimated that even differences in propagation distance from the same point on the cloud surface may account for relative time delays of .1 sec., without increasing the apparent angular size beyond that observed. Since the largest correlation times (pulse lengths) emitted would be about .002 sec., according to the coherent model, then the random effects of time delays of this order of magnitude would totally dephase the Fourier components and, according to the central limit theorem, yield Gaussian statistics by the time the radiation reached earth. Interstellar scattering which redistributes the direction of wave propagation over the solid angle defined by the apparent diameter also implies that radiation from different regions of the cloud with slightly different turbulent velocities (within a linewidth) will be received simultaneously. This will even more certainly cause the spectrum which theoretically should appear as an envelope of almost discrete lines, to appear as continuous across the line. Thus, from what is generally believed about the magnitude of interstellar scattering, it is reasonable to expect that any non-Gaussian characteristics of radiation from astrophysical OH masers could not be detected.

d) Summary of Observational Evidence

In summary, then, besides many theoretical advantages deriving from not imposing arbitrary restraints on the maser radiation, our coherence hypothesis seems to account for the observational data better than previous theories in the following ways:

1). It explains the broadening seen in several class I OH sources that have been carefully measured for the most intense two or three lines, with W3 providing a suggestive example of broadening proportional to $E^2_{rms}$ in five neighboring lines.

2). It allows for the existence of lines much narrower than the minimum that could likely be obtained by unsaturated narrowing, given the small range of intensities involved.

3). Similarly, it explains how lines of fairly high intensity can have a width approximately that reached during unsaturated narrowing, even while saturated. (There is a balance between the broadening tendency of saturation and the narrowing tendency of the onset of coherence.)

4). It is completely consistent with effects regarding relative apparent diameters believed to derive from interstellar scattering of the emitted radiation, and the measured statistics of that radiation. It predicts actual maser sizes closer to actual VLBI sizes, thus much smaller than for previous theories.

Appendix A

Using the notation of Section III for the Maxwell-Bloch equations we let $P = Y + iX$ here.

Then the $\dot{X}$ and $\dot{Y}$ equations become;

$$\dot{P} = -i\delta P + pEZ - P/T_2. \tag{A1}$$

The $\dot{Z}$ equation becomes:

$$\dot{Z} = -\frac{p}{2}(E^*P + P^*E) + \lambda(1 - Z) - \frac{(Z - Z_o)}{T_1} \tag{A2}$$

If $Z' = \left(\lambda + \frac{Z_o}{T_1}\right) / \left(\lambda + \frac{1}{T_1}\right)$, and $S(\delta) = \left(\frac{1}{T_2} + i\delta\right)$ then Fourier analyzing (A1) and (A2) gives, respectively:

$$\rho(\omega, \delta) = \frac{p\int_{-\infty}^{\infty} E(\omega - \nu)\, Z(\nu, \delta)\, d\nu}{\sqrt{2\pi}\, S(\delta - \omega)}, \tag{A3}$$

and

$$Z(\nu, \delta) = \frac{\left(\lambda + Z_o/T_1\right)}{\left(\lambda + 1/T_1\right)}\, \delta(\nu) - \frac{p}{2\sqrt{2\pi}\left(\lambda + \frac{1}{T_1} - i\nu\right)} \int_{-\infty}^{\infty} \left[E^*(\omega - \nu) P(\omega, \delta) + E(\omega + \nu) P^*(\omega, \delta)\right] d\omega. \tag{A4}$$

Here $E(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E(\omega)\, e^{-i\omega t}\, d\omega$, and $\omega = 0$ represents resonant frequency components. By substituting for $P(\omega, \delta)$ in (A4) we obtain:

$$Z(\nu, \delta) = \left(\frac{\lambda + Z_o/T_1}{\lambda + 1/T_1}\right) \delta(\nu) - \frac{p^2}{4\pi} \frac{1}{\left(\lambda + \frac{1}{T_1} - i\nu\right)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\nu' d\omega \times$$

$$\left[\frac{E^*(\omega-\nu)\,E(\omega-\nu')}{S(\delta-\omega)} + \frac{E^*(\omega+\nu')\,E(\omega+\nu)}{S(\omega-\delta)}\right] Z(\nu',\delta), \tag{A5}$$

since $Z(\nu',\delta) = Z^*(-\nu',\delta)$, and $S^*(\delta-\omega) = S(\omega-\delta)$. Equation (A5), then, is an exact integral equation for $Z(\nu,\delta)$ which can be iterated to give a series of terms containing increasing powers of $E^2(\omega)$, in a manner similar to Appendix B is Goldreich, et al. (1973a). The zeroeth order answer for $Z(\nu,\delta)$ is just $[(\lambda+Z_o/T_1)\,/\,(\lambda+1/T_1)]\,\delta(\nu)$, as one would expect with no field present. The correlation $< E(\omega)\,E^*(\omega+\nu)\,z(\nu) >$ to zeroth order is then just $E^2(\omega)\,(\lambda+Z_o/T_1)\,/\,(\lambda+1/T_1)$, where $E(\omega)\,E^*(\omega) = E^2(\omega)$, since assuming Gaussian statistics for the field gives, in general,

$$\int_{-\infty}^{\infty} E(\omega+\nu)\,E^*(\omega)\,d\nu = E^2(\omega). \tag{A6}$$

After one iteration for $Z(\nu,\delta)$,

$$\langle E(\omega)\,E^*(\omega+\nu)\,Z(\nu\,\delta)\rangle = E^2(\omega)\left(\frac{\lambda+Z_o/T_1}{\lambda+1/T_1}\right)\delta(\nu) - \frac{p^2}{4\pi}\times\frac{1}{\left(\lambda+\frac{1}{T_1}-i\nu\right)}\times\frac{\left(\lambda+Z_o/T_1\right)}{\left(\lambda+1/T_1\right)}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\nu'\,d\omega' \times$$

$$\left[\frac{E^*(\omega'-\nu)\,E(\omega'-\nu')\,E(\omega)\,E^*(\omega+\nu)}{S(\delta-\omega)} + \frac{E^*(\omega'+\nu')\,E(\omega'+\nu)\,E(\omega)\,E^*(\omega+\nu)}{S(\omega-\delta)}\right]\delta(\nu') \tag{A7}$$

Again assuming Gaussian statistics initially for the field we can evaluate the products of E in the brackets in by taking all paired combinations. The first term in brackets becomes (noting the $\delta(\nu')$ and using (A6):

$$\left[\frac{E^2(\omega')\,E^2(\omega)\,\delta(\nu)\,\delta(\nu')}{S(\delta-\omega')} + \frac{E^2(\omega)\,E^2(\omega+\nu)\,\delta(\nu')\,\delta(\omega'-\nu-\omega)}{S(\delta-\omega')}\right] \tag{A8}$$

Similarly, the second term becomes:

$$\left[\frac{E^2(\omega') E^2(\omega)\,\delta(\nu)\,\delta(\nu')}{S(\omega'-\delta)} + \frac{E^2(\omega)\,E^2(\omega+\nu)\,\delta(\omega'-\omega)\,\delta(\nu')}{S(\omega'-\delta)}\right] \tag{A9}$$

Substituting (A8) and (A9) into (A7) and doing the $\omega'$ integral we get:

$$\langle E(\omega)\,E^*(\omega+\nu)\,Z(\nu,\delta)\rangle = \frac{\lambda + Z_o/T_1}{\lambda + 1/T_1} \times E^2(\omega) \times$$

$$\left\{\delta(\nu)\left(1 - \frac{p^2 E^2(\delta)}{2(\lambda+1/T_1)}\right) + \frac{p^2 E^2(\omega+\nu)}{\lambda + \frac{1}{T_1} - i\nu} \times \left[\frac{1}{S(\delta-\omega-\nu)} + \frac{1}{S(\omega-\delta)}\right]\right\} \tag{A10}$$

While the $E^2(\delta)$ term just indicates the beginning of saturation depleting the population inversion, the $E^2(\omega+\nu)$ terms clearly show how frequencies within a range $\pm\nu \sim (\lambda + \frac{1}{T_1})$ about $\omega$ are becoming correlated with each other through oscillation developing in Z. This effect is large at saturation when $(p^2 E^2(\omega))/(\lambda + 1/T_1) \cong 1$.

We can also use eqn. (A5) to recalculate the auto-correlation function for Z, viz. $\langle Z(t+\tau)\,Z(t)\rangle$, as did Goldreich, et al.. For this we need only $\langle Z(\nu)\,Z^*(\nu'')\rangle$ to 2nd order, an we will only look here at the term where $\nu = \nu''$, containing 4th powers of E, where a correction to Goldreich, et al., eqn. (B9) is needed. The relevant term is:

$$\left(\frac{\lambda + Z_o/T_1}{\lambda + 1/T_1}\right)^2 \left(\frac{p^2}{4\pi}\right)^2 \times \frac{1}{\lambda + \frac{1}{T_1} - i\nu} \times \frac{1}{\lambda + \frac{1}{T_1} - i\nu'} \times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\omega'\,d\rho \times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\omega\,d\nu'$$

$$\left[\frac{E^*(\omega-\nu)E(\omega-\nu')\delta(\nu')}{S(\delta-\omega)} + \frac{E^*(\omega+\nu')\,E(\omega+\nu)\,\delta(\nu')}{S(\omega-\delta)}\right] \times \tag{A11}$$

$$\left[\frac{E(\omega'-\nu'')\,E^*(\omega'-\rho)\,\delta(\rho)}{S^*(\delta-\omega')} + \frac{E(\omega'+\rho)\,E^*(\omega'+\nu'')\,\delta(\rho)}{S^*(\omega'-\delta)}\right]$$

Multiplying out the brackets we get:

$$\left\{ \frac{E^*(\omega-\nu)\,E(\omega)\,E(\omega'-\nu'')\,E^*(\omega')}{S(\delta-\omega)\,S^*(\delta-\omega')} + \frac{E^*(\omega)\,E(\omega+\nu)\,E(\omega'-\nu'')\,E^*(\omega')}{S(\omega-\delta)\,S^*(\delta-\omega')} \right.$$

$$\left. + \frac{E^*(\omega-\nu)\,E(\omega)\,E(\omega')\,E^*(\omega'+\nu'')}{S(\delta-\omega)\,S^*(\omega'-\delta)} + \frac{E^*(\omega)\,E(\omega+\nu)\,E(\omega')\,E^*(\omega'+\nu'')}{S(\omega-\delta)\,S^*(\omega'-\delta)} \right\} \tag{A12}$$

The terms of interest where $\nu = \nu''$ are:

$$\left\{ \frac{E^2(\omega)\,E^2(\omega-\nu)\,\delta(\nu-\nu'')\,\delta(\omega-\omega')}{S(\delta-\omega)\,S^*(\delta-\omega)} + \frac{E^2(\omega+\nu)\,E^2(\omega)\,\delta(\nu-\nu'')\,\delta(\omega'-\omega-\nu)}{S(\omega-\delta)\,S^*(\delta-\omega-\nu)} \right.$$

$$\left. + \frac{E^2(\omega-\nu)\,E^2(\omega)\,\delta(\nu-\nu'')\,\delta(\omega'-\omega+\nu)}{S(\delta-\omega)\,S^*(\omega-\nu-\delta)} + \frac{E^2(\omega)\,E^2(\omega+\nu)\,\delta(\omega-\omega')\,\delta(\nu-\nu'')}{S(\omega-\delta)\,S^*(\omega-\delta)} \right\} \tag{A13}$$

respectively. Again, we see that many Fourier components of $E^2$ (e.g. $E^2(\omega)\,E^2(\omega+\nu)$) contribute to the $\frac{\delta(\nu-\nu'')}{\Gamma^2+\nu^2}$ term in $\langle Z(\nu)\,Z^*(\nu'')\rangle$, where here $\Gamma = \lambda + \frac{1}{T_1}$, and not just $f(\omega_o)$ in the notation of Goldreich, et al.

## Appendix B

We will demonstrate here that the rate of growth of pulse energy in the case where $\tau >> T_2^*$ (but $\tau << T_2$) is quite sufficient to overcome energy losses from $T_2$ being finite. From our solution the expression for the rate of increase of pulse energy is:

$$\frac{dE_p}{dz} = 4\lambda T_2^* N\hbar\omega_o K_o e^{A(z-z_o)}$$

McCall and Hahn (1969) have calculated the rate of energy loss due to $T_2$ in the case where one has hyperbolic secant pulses, i.e. where $K_o >> \pi/2$ here. They get:

$$\frac{dE_p}{dz} = - \frac{2\sqrt{\pi}\, N\hbar\omega_o}{3} \left( \frac{T_2^*}{T_2} \right)$$

If $T_2 \cong \lambda^{-1}$ as is likely in the astrophysical case then we see that even for $z = z_o$, the energy lost be the dephasing action of the pump mechanism is small compared to the rate of growth, and it gets relatively smaller as amplification occurs.

In the case where $\tau << T_2^*$ there is certainly no problem if $\lambda << T_2^{*-1}$.

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